University of Bath

Department of Mechanical Engineering

Control and Mechatronics Practical Project Report

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Summary

During this project, Lego parts and hardware were provided to construct a closed loop position control system. This report outlines the importance of controllers in engineering before highlighting any novel changes that were made to the proposed layout. The ultrasonic sensor's performance was tested using a controlled environment, leading to an offset of 2.1 mm and a gain of 1.032 on the sensor signal. An investigation into different filters was then carried out leading to a FIR filter being selected due to its low phase lag and good filtering properties. The system was then modelled by combining a FBD with Newton's Second Law to represent the plant dynamics where proportional and derivative gains of 2.24 and 0.668 were derived. Subsequently, these gains were tested in a simulated environment that considered the servo reaction speed, rail length limitations and phase lag. The gains that were analytically determined were then automatically tuned to achieve the best possible response in this simulated environment. The tuned controller was tested on the real system but initially produced suboptimal results. As a result, an integral gain was introduced to improve performance. The effects of friction and sensor limitations were also explored, and the performance of the optimized controller achieved a rise time of 1.05 seconds, an overshoot of 13.3%, a steady-state error of 0.003 m, a gain margin of 29 dB, and a phase margin of 75 degrees.

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Introduction

Control systems are vital to the advancement of technology in engineering, enabling the regulation of dynamic systems. Common types include position and speed controllers. At Green Bath Racing (GBR), selecting an optimal speed controller is essential for smooth driver responses and efficient power delivery. When designing and tuning controllers, factors such as operating conditions and desired performance must be carefully considered. For GBR's speed controller, the goal is to align the system's rise time with the car's most efficient acceleration profile while avoiding oscillatory speed responses. In contrast, for a position controller balancing a cart on rails, minimising rise time is crucial, and slight oscillations may be more acceptable.

This project designs a closed-loop control system to balance a cart on rails using an ultrasonic sensor, an RC servomotor, and an Arduino to implement the controller. Through modelling, simulation, and practical testing, this coursework explores key control engineering principles and the critical role controllers play in achieving desired system behaviour.

The mechanical arrangement of the cart balancing on rails may be visualised in Figure 1:

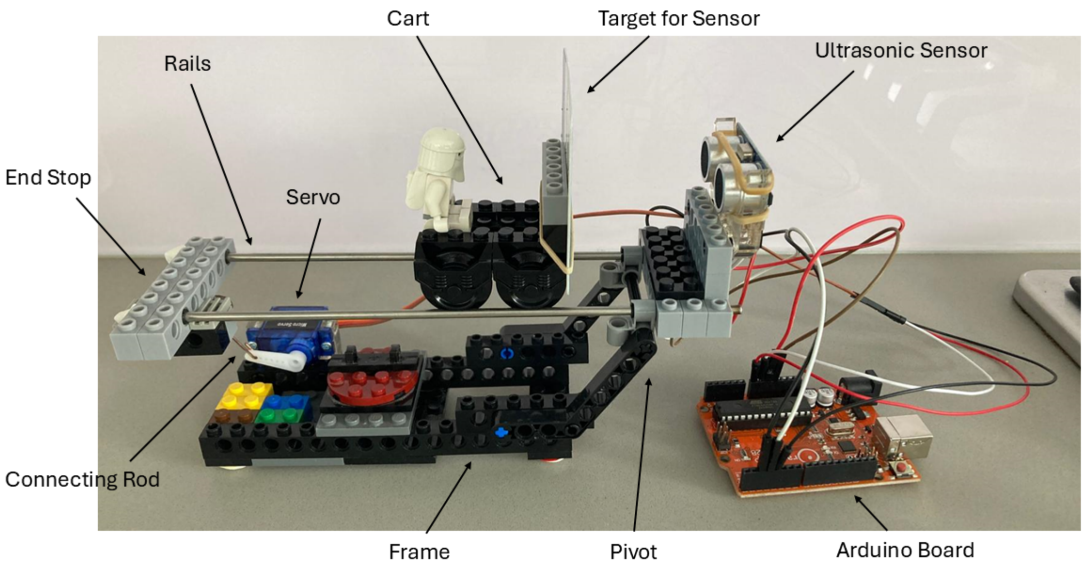

Figure 1 shows the mechanical arrangement of the cart balancing on rails as well as the technical hardware used in this project.

The following mechanical changes were implemented to improve the performance of the system:

- Centre of mass was shifted further back
  - To prevent forward tilting when the cart is positioned in the front
- Blue tack was applied on four points of contact with the ground
  - To improve stability and reduce vibrations absorbed by the system
- Rails were fixed to the end stop
  - To prevent sliding of the rails during testing
- Ultrasonic sensor was constrained to the pivot
  - To prevent sliding of the sensor during testing
- WD40 was applied to the cart wheels
  - To reduce the effect of friction

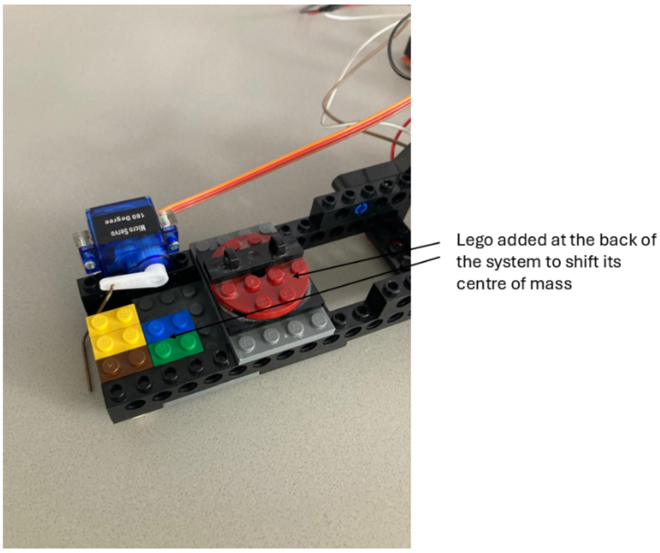

Figure 2 shows the weight that was added to the rear end of the system to shift the centre of mass.

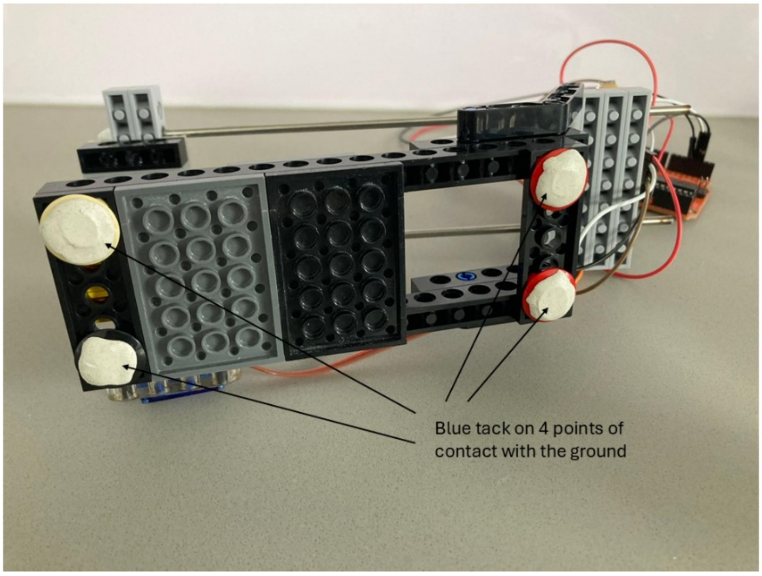

Figure 3 shows the blue tack that was applied on the bottom of the system.

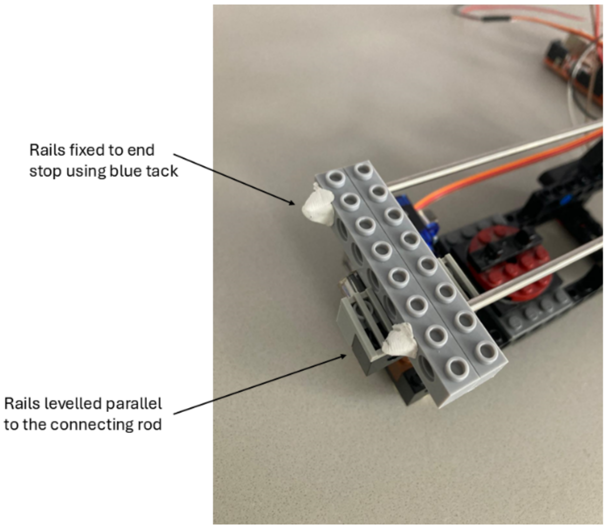

Figure 4 shows the constraining of the rails to the end stop, as well as the levelling of the rails to be parallel with the connecting rod.

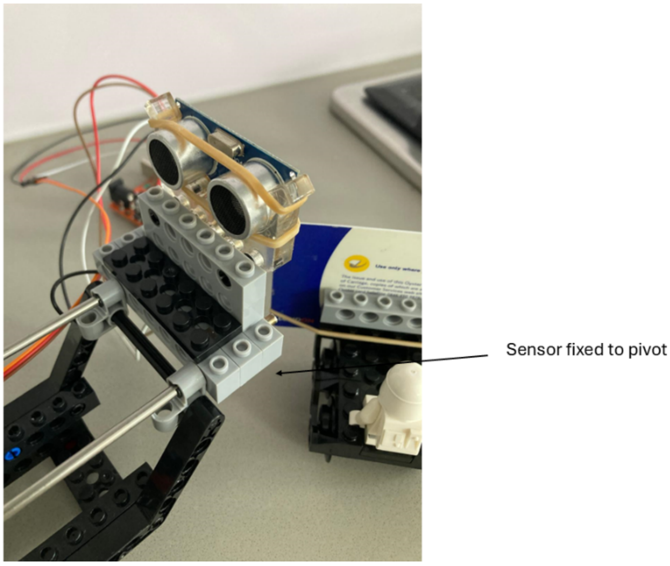

Figure 5 shows how the ultrasonic sensor was fixed to the pivot such that it didn't slide off from the rails.

The HC-SR04 ultrasonic sensor operates on a 5V DC power supply with a minimum trigger pulse of 10 microseconds. It has a measuring range of 2 to 400 cm with a resolution of 0.3 cm. Distance is determined by calculating the time it takes for sound waves to travel to and from an object, based on the speed of sound.

The RC servomotor also requires a 5V DC power supply. It can range 90 degrees in 60ms and provides a maximum torque of 14Ncm.

Equation 1 derives the rate of the RC servomotor.

$$Speed = \frac{\theta}{T} = \frac{90}{0.06} = 1500\frac{\circ}{s} = 26.18\, rad/s$$

The microcontroller is a Kona328 Arduino board with 14 digital input/output pins, 6 analog inputs, a 16 MHz crystal oscillator, a USB port, a power jack, an ICSP header and a reset button.

Results

Calibration and Filtering

The following procedure was used to calibrate the ultrasonic sensor:

- Actual distances measured using a set square and a ruler
- Distances measured using the ultrasonic sensor
- Measured and actual distances plotted on the y and x axes respectively
- Linear fit applied to recorded data

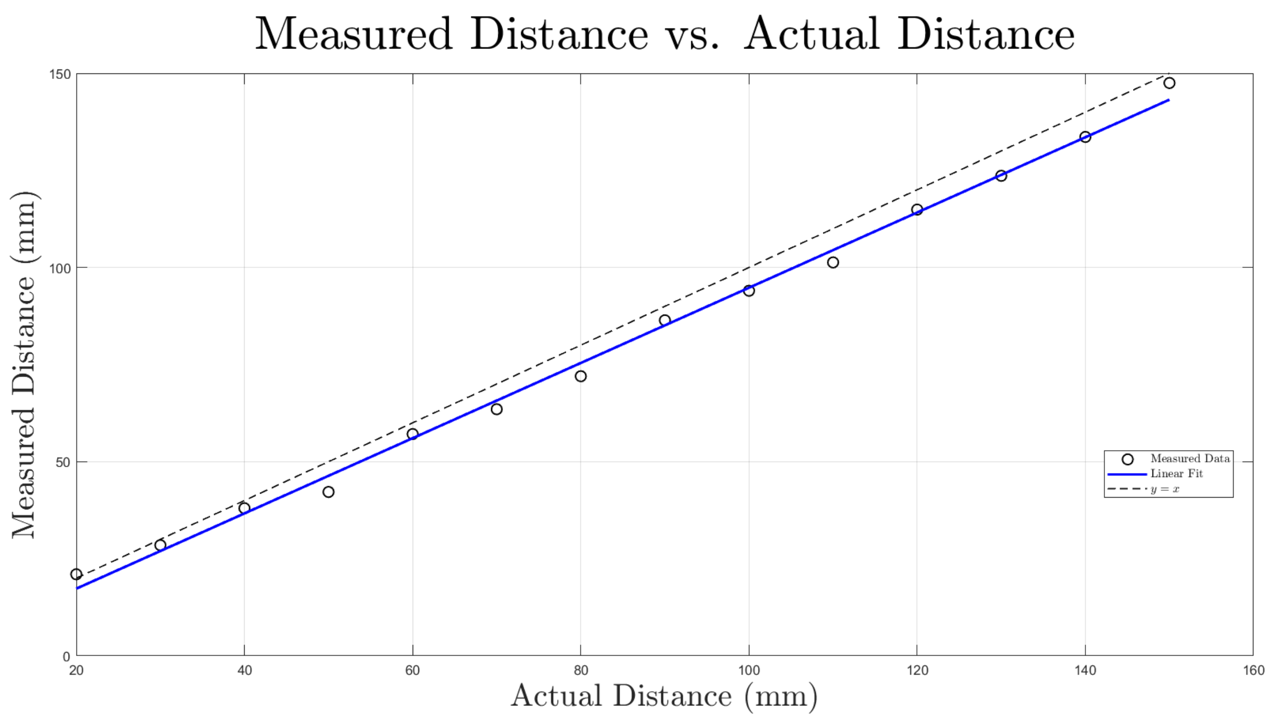

Figure 6 shows the linear fit that was applied to measured distances. The equation of the line is y = 0.969x – 0.21. The plot also compares the sensor's performance to a perfect response of line equation y = x.

A Fast Fourier Transform (FFT) will be applied to sampled data to determine a cut off frequency and is plotted on figure 7:

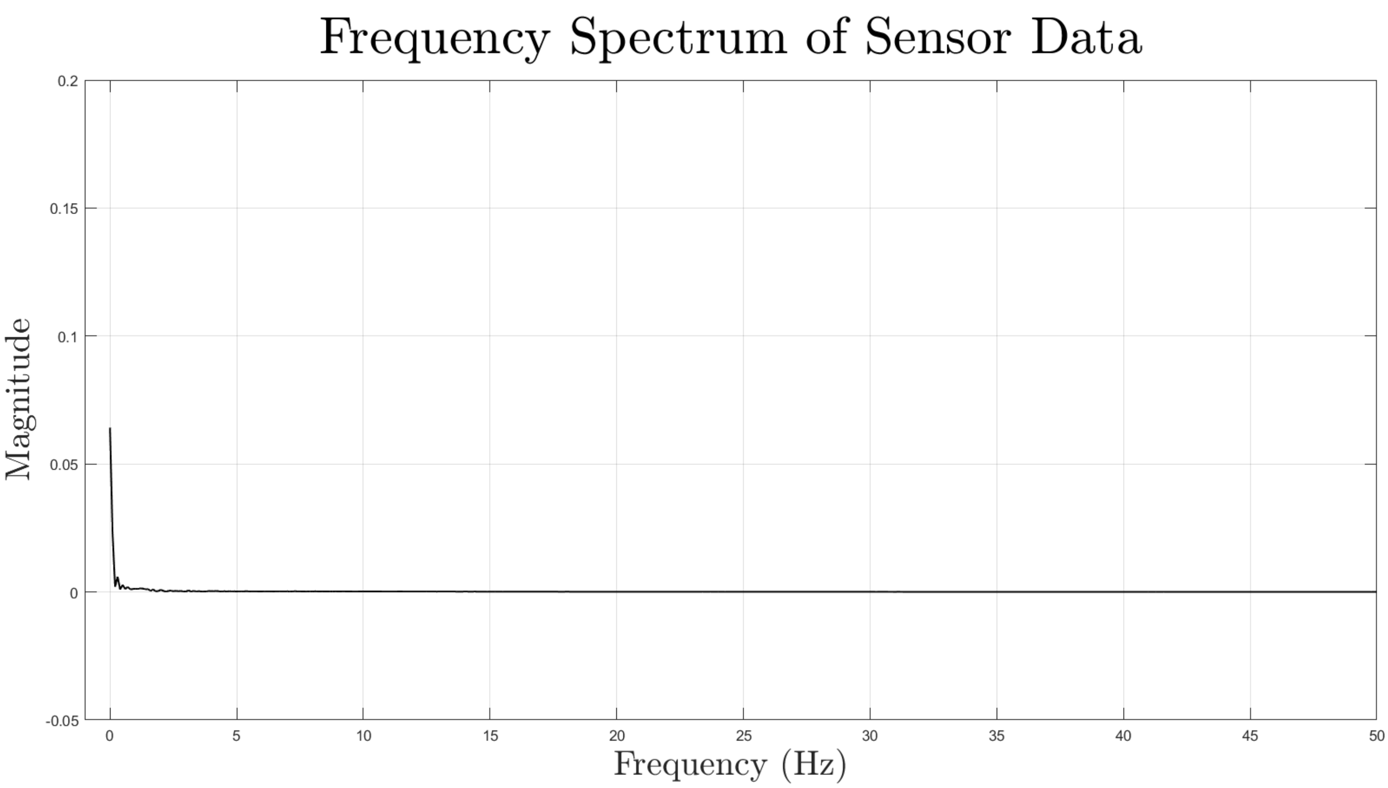

Figure 7 shows the FFT of sampled data. This plot shows that most frequencies detected by the ultrasonic sensor range from 0 to 5 HZ. Therefore, a cut off frequency of 20Hz will be selected for subsequent filters.

Three different low pass filters were explored for this task:

- Analytical filter
- Finite Impulse Response (FIR)
- Infinite Impulse Response (IIR)

For a cut off frequency of 20 Hz at a sampling frequency of 200 Hz, the continuous time transfer function is as follows, given that we are using the backwards Euler method:

Equation 2 is the derivation of the transfer function for a continuous time low pass filter.

$$\tau = \frac{1}{2 \times \pi \times f_c} = \frac{1}{40\pi}$$

$$Filter = \frac{1}{\tau s + 1}$$

where

$$s = \frac{1 - z^{-1}}{\Delta}$$

$$Filter = \frac{z}{2.59z - 1.59}$$

The following figures compare in detail the effect of each filter on a signal.

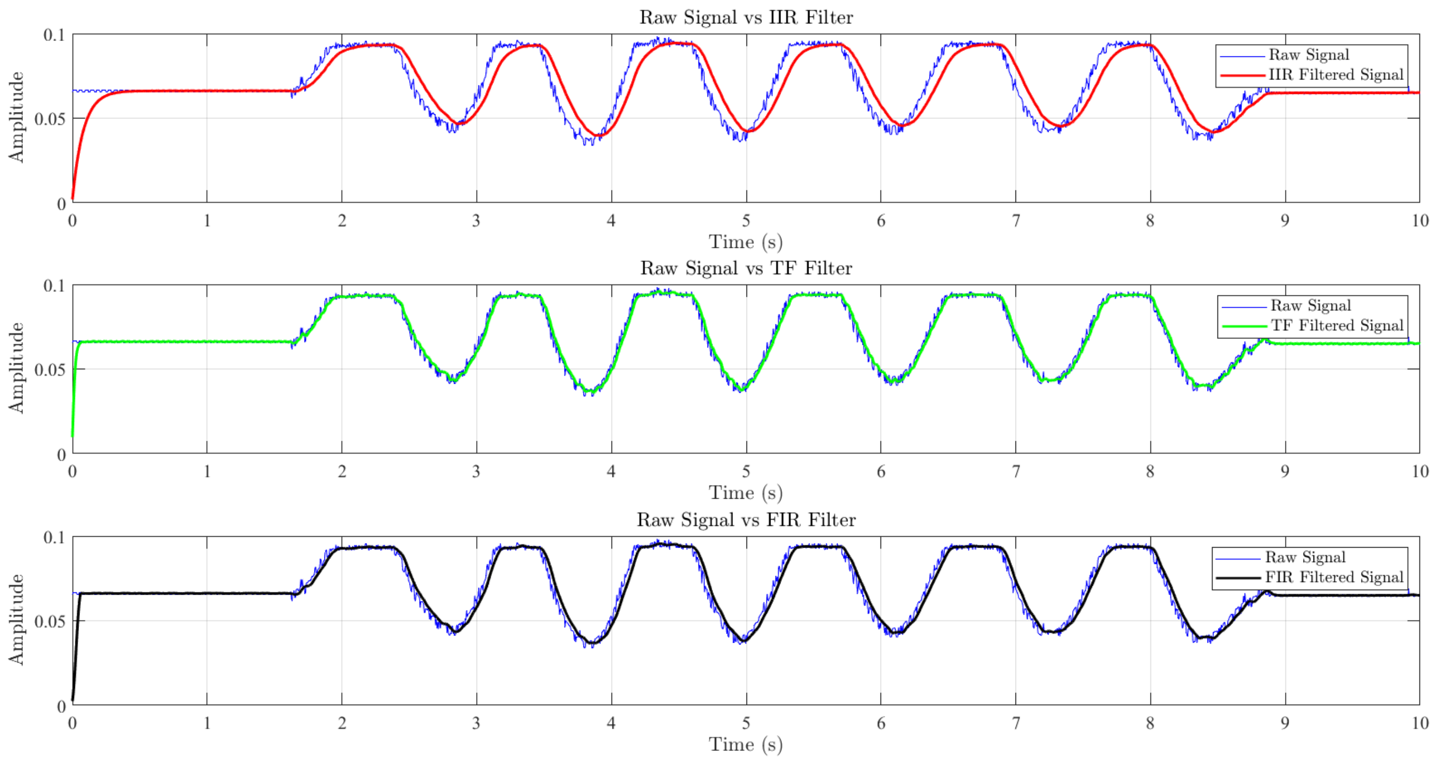

Figure 8 compares the three discussed filters on sample data.

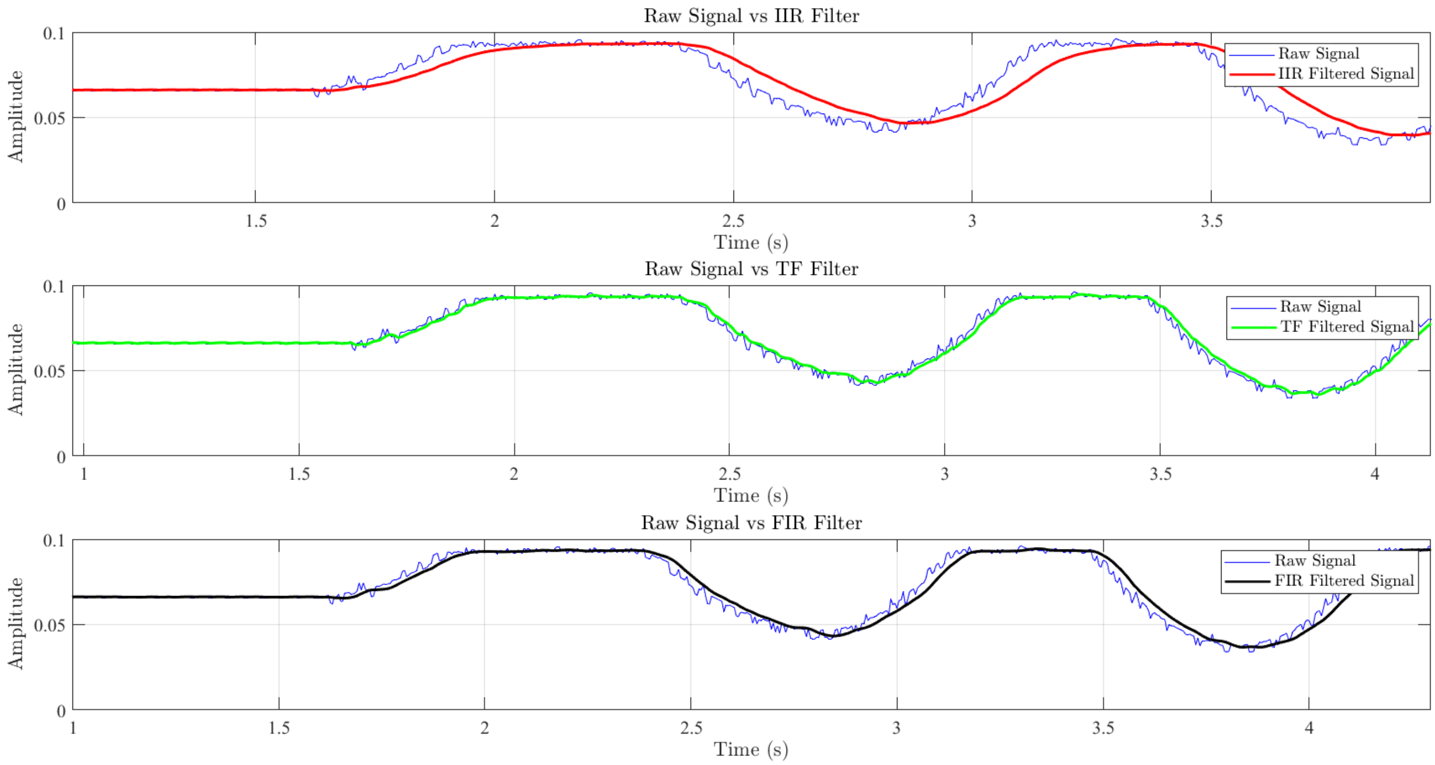

Figure 9 compares a close up of the three filters to gain a better understanding of each effect of the filters.

Modelling and Controller Design

The following free body diagram is essential to derive a transfer function representative of the system.

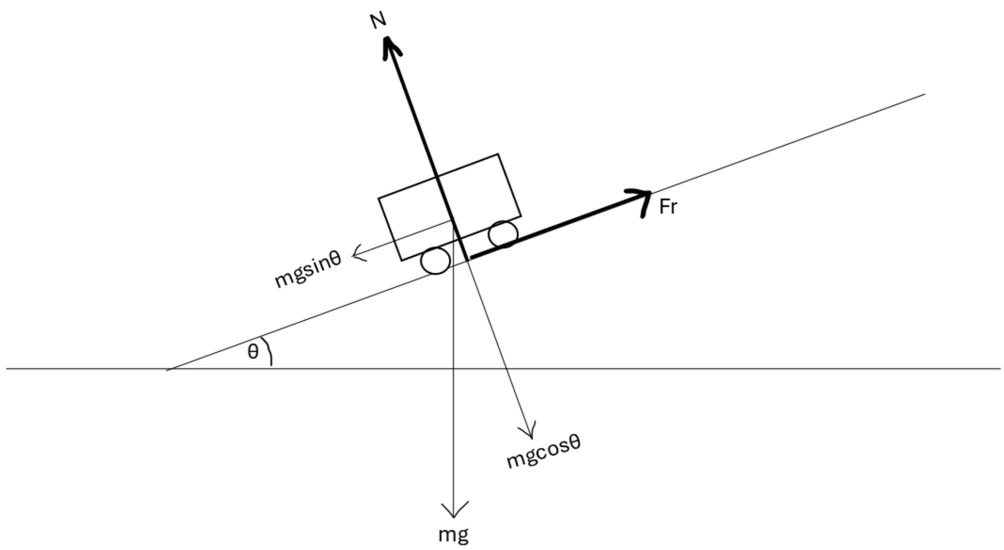

Figure 10 depicts the Free Body Diagram of the cart on its rails

From figure 10, Newton's Second Law will be applied to determine the transfer function relating the rail angle to the position of the cart.

Equation 3 shows the derivation of the transfer function relating the rail angle to the position of the cart

$$F = m\ddot{x}$$

substituting F for the component of the cart's weight that acts parallel to the slope

$$mgsin\theta = m\ddot{x}$$

cancelling mass terms on both sides of the equation

$$gsin\theta = \ddot{x}$$

using small angle approximation where sinθ = θ

using the Laplace transform

$$Xs^2 = g\theta$$

$$\frac{X}{\theta} = \frac{g}{s^2}$$

This transfer function may now be coupled with assumptions as well as equations 4 and 5 to estimate initial gains for the system. The following were assumed:

- Proportional Derivative controller will be used
- Natural frequency of the system is 5 rad/s
- Damping ratio is 0.7
- Feedback sensor is perfect

Equation 4 is the derivation of the closed loop transfer function.

$$\frac{CG}{1 + CGH}$$

$$\frac{(k_p + k_d s)g/_{s^2}}{1 + 1 \times (k_p + k_d s)g/_{s^2}}$$

$$\frac{g(k_p + k_d s)}{s^2 + k_d s + k_p}$$

Comparing the denominator to its standard form yields:

Equation 5 is the standard form of a second-order characteristic equation.

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Therefore, given the assumptions made, the estimated proportional gain is 2.24 and the derivative gain is 0.668.

Simulation

Applying the PD controller that was derived from equations 4 to 5, yields the following response:

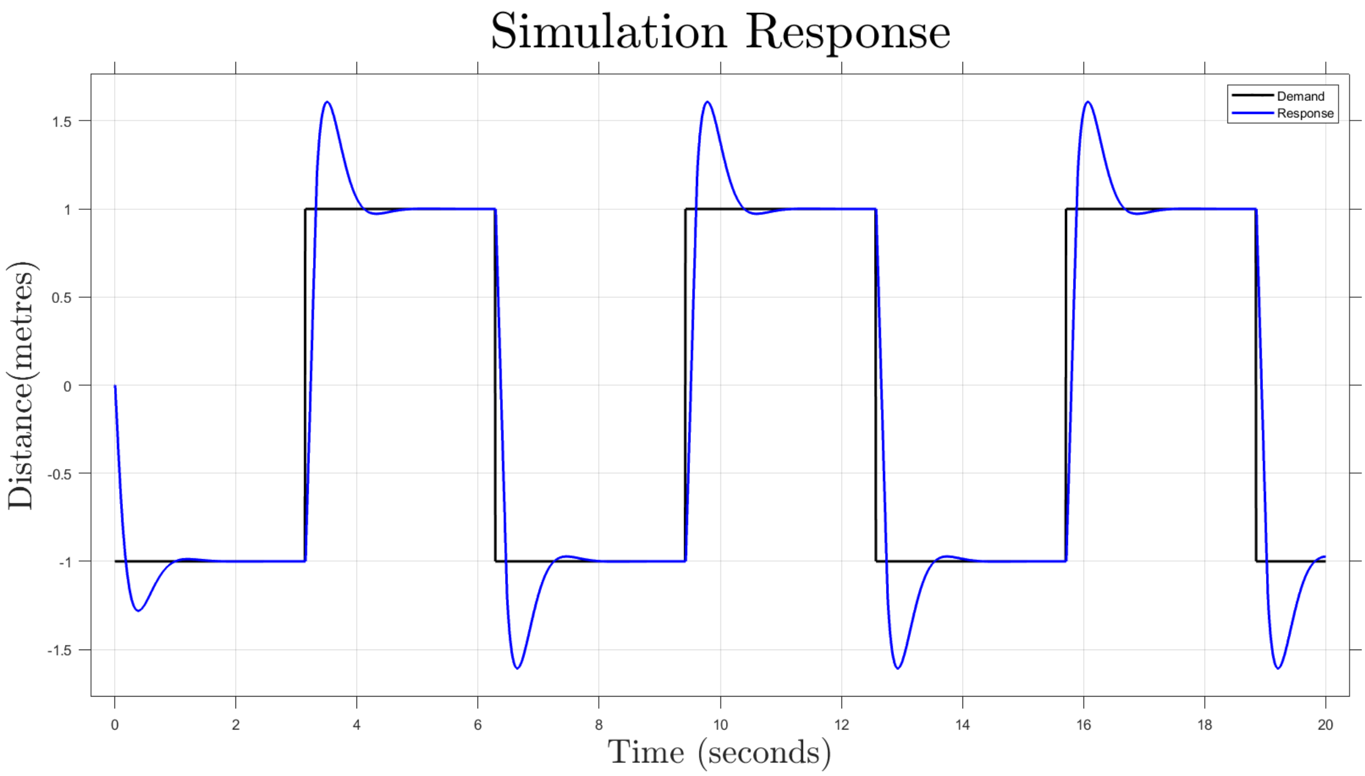

Figure 11 shows the Proportional Derivative controller response from derived gains from equations 4 to 6. The response has a 60.6% overshoot and a 1.05 second rise time with 0m steady state error.

Figures 12, 13 and 14 show how considering the servo speed, rail length and phase lag affects the response.

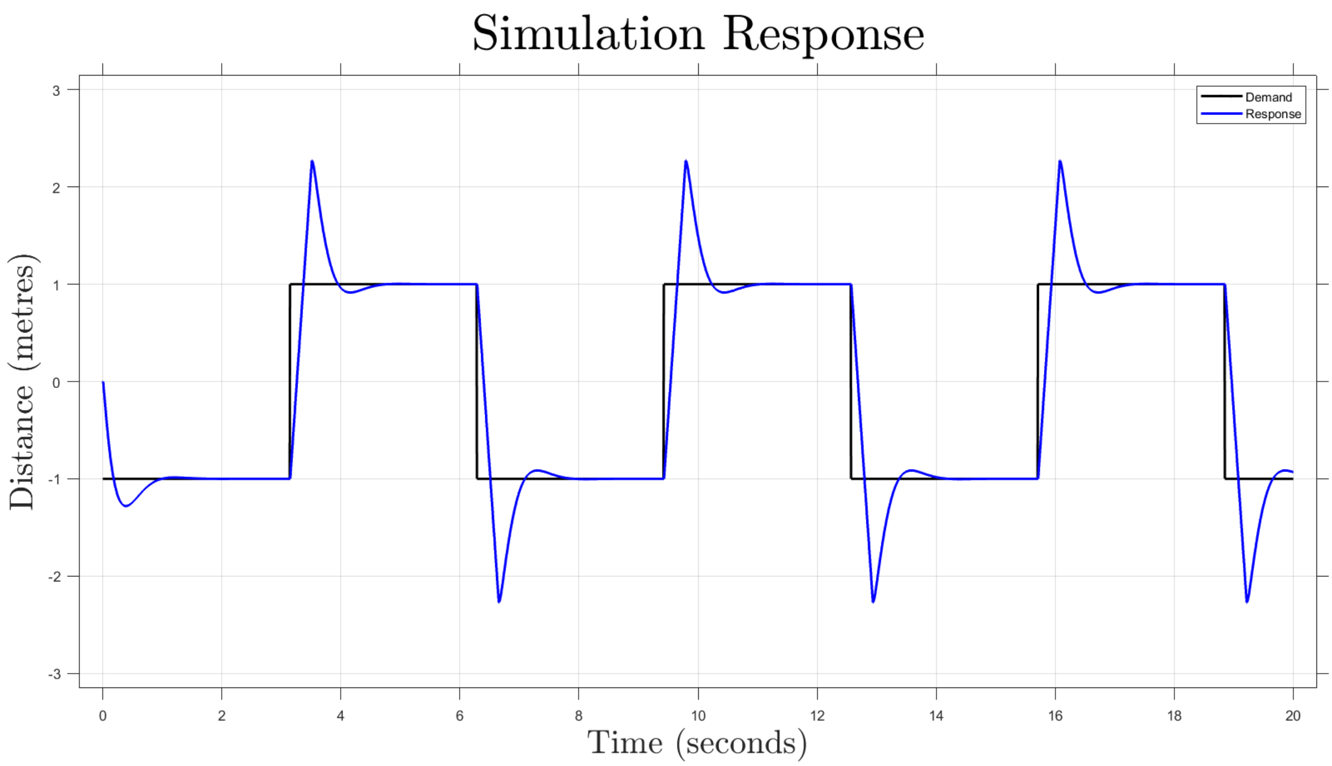

Figure 12 depicts how the physical limitations affect the response of the controller. Considering the servo speed limitation, the response overshoots by 127% and its rise time is 1.42 seconds with 0m steady state error.

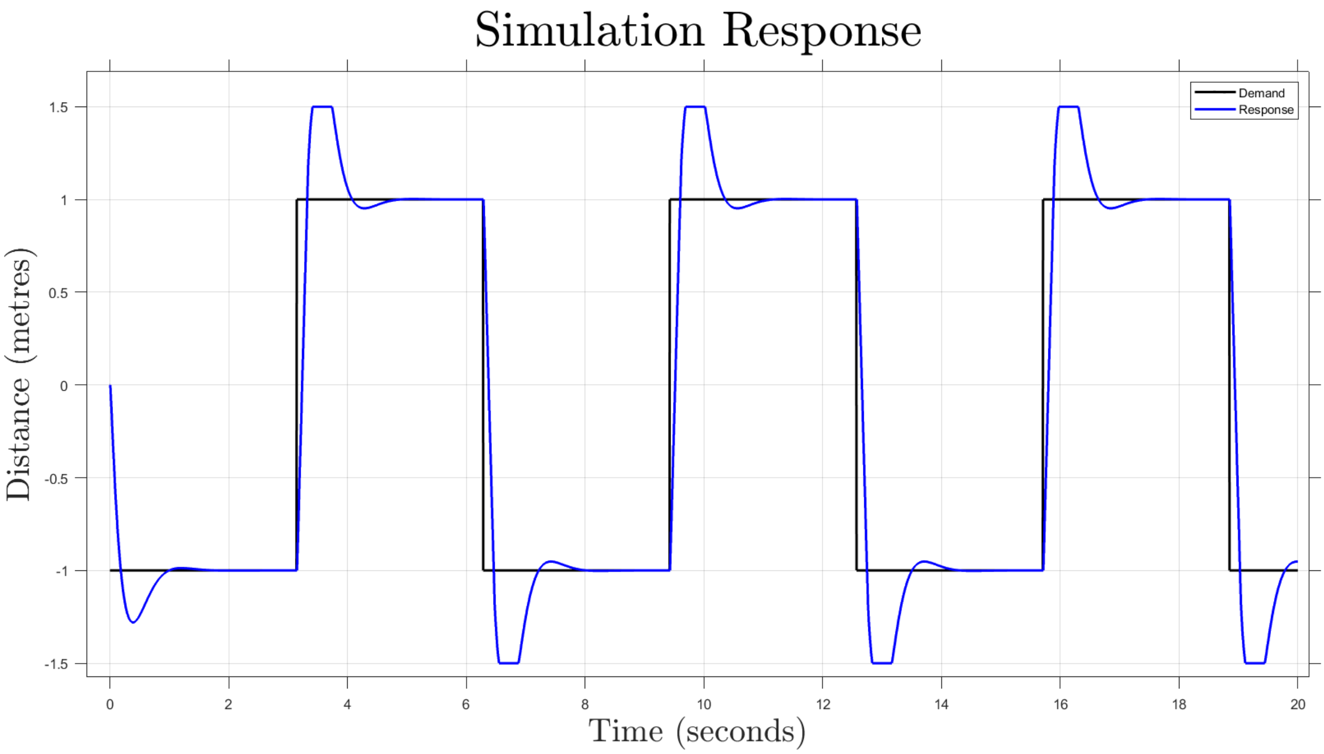

Figure 13 shows how considering the physical limit of the system, for example the rail lengths, affects the response. Limiting the rail length to 1.5 metres would provide an overshoot of 50% and a rise time of 1.55 seconds with 0m steady state error.

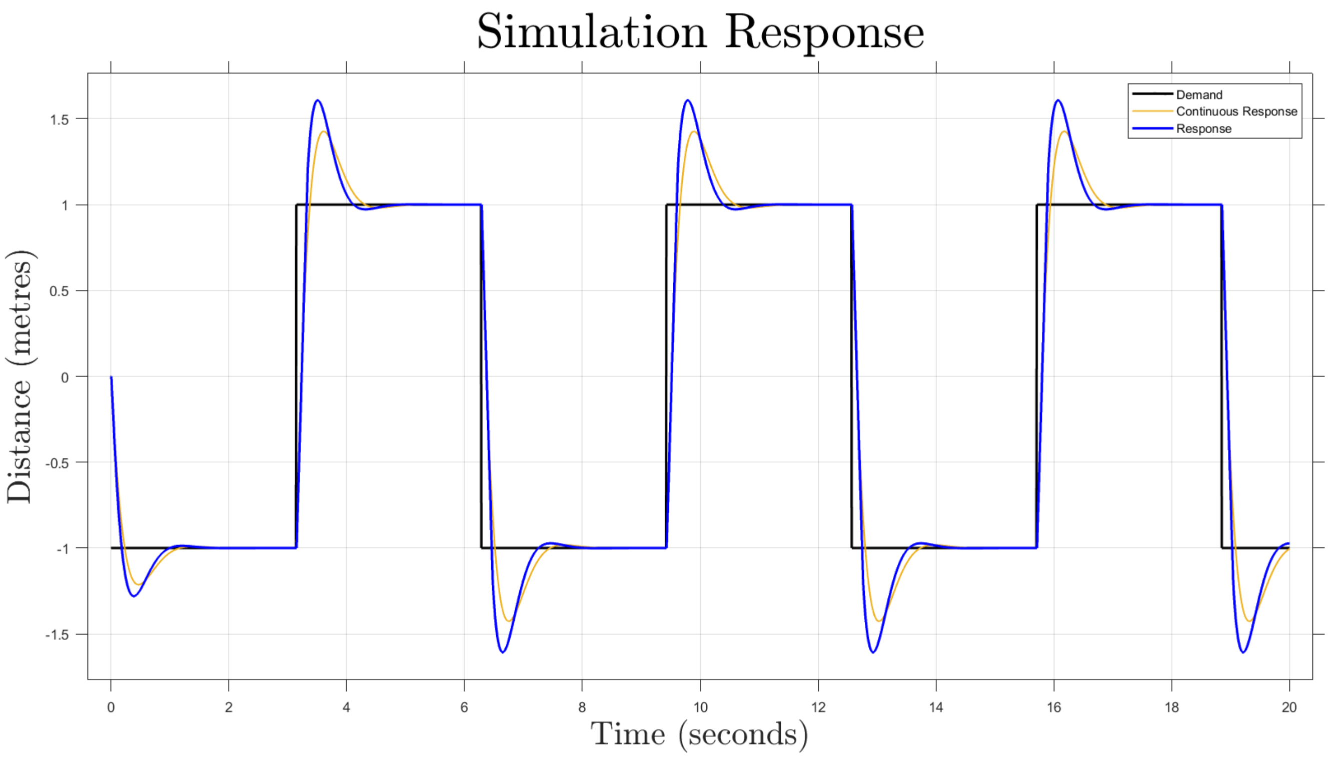

Figure 14 shows how considering the phase lag introduced by real time filter will affect the response. In the ideal case with no phase lag, the overshoot with this proportional derivative controller would be 42.6%, and the rise time would be 1.16 seconds with 0m steady state error.

Tuning the gains that were derived from equations 4 to 6 yields the following response:

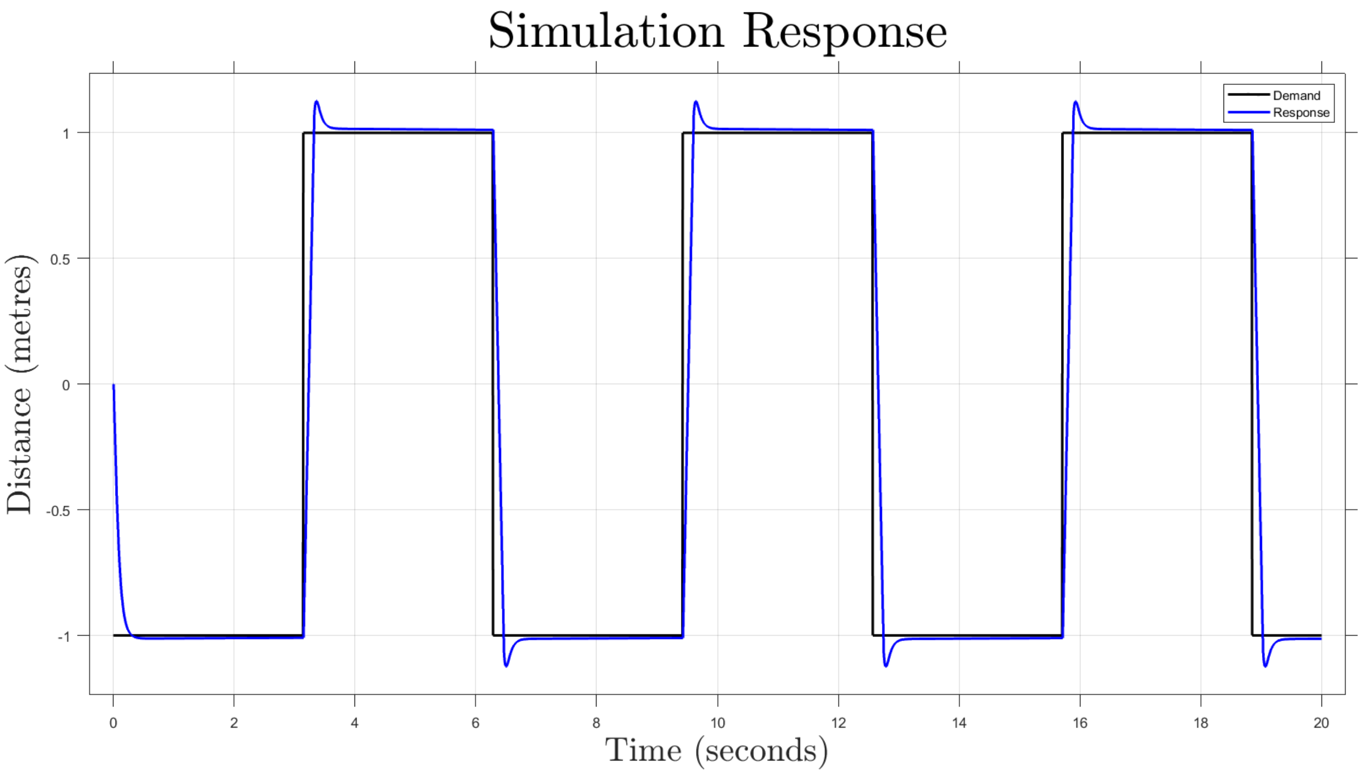

Figure 15 shows the response after tuning the proportional and derivative gains to 0.1 and 0.9 respectively. The response yields an overshoot of 12.6% and a rise time of 0.52 seconds with a 0m steady state error

System Testing

Figures 16, 17 and 18 plot the response of the controller derived from the model, from the simulation and from trial and error respectively. Figure 19 explores the effect of neglecting friction in the system and figure 20 depicts the trial-and-error process to achieve the best possible controller given a near zero friction in the system. The last figure plots a bode plot to illustrate the overall stability of the final controller.

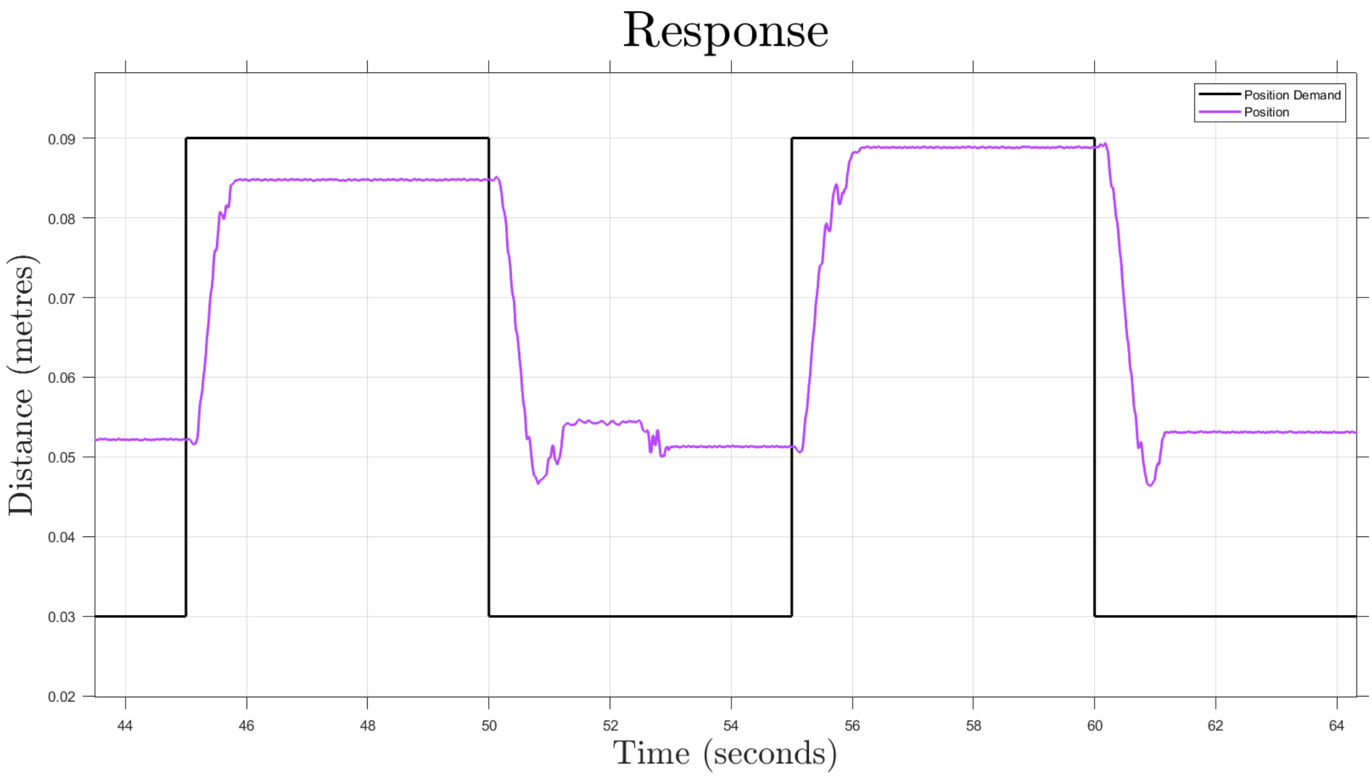

Figure 16 shows the online response with a proportional gain of 2.24 and a derivative gain of 0.668, as determined from the modelling of the system. The response yields a 73 % overshoot and a rise time of 1.08 seconds with a 0.023m steady state error.

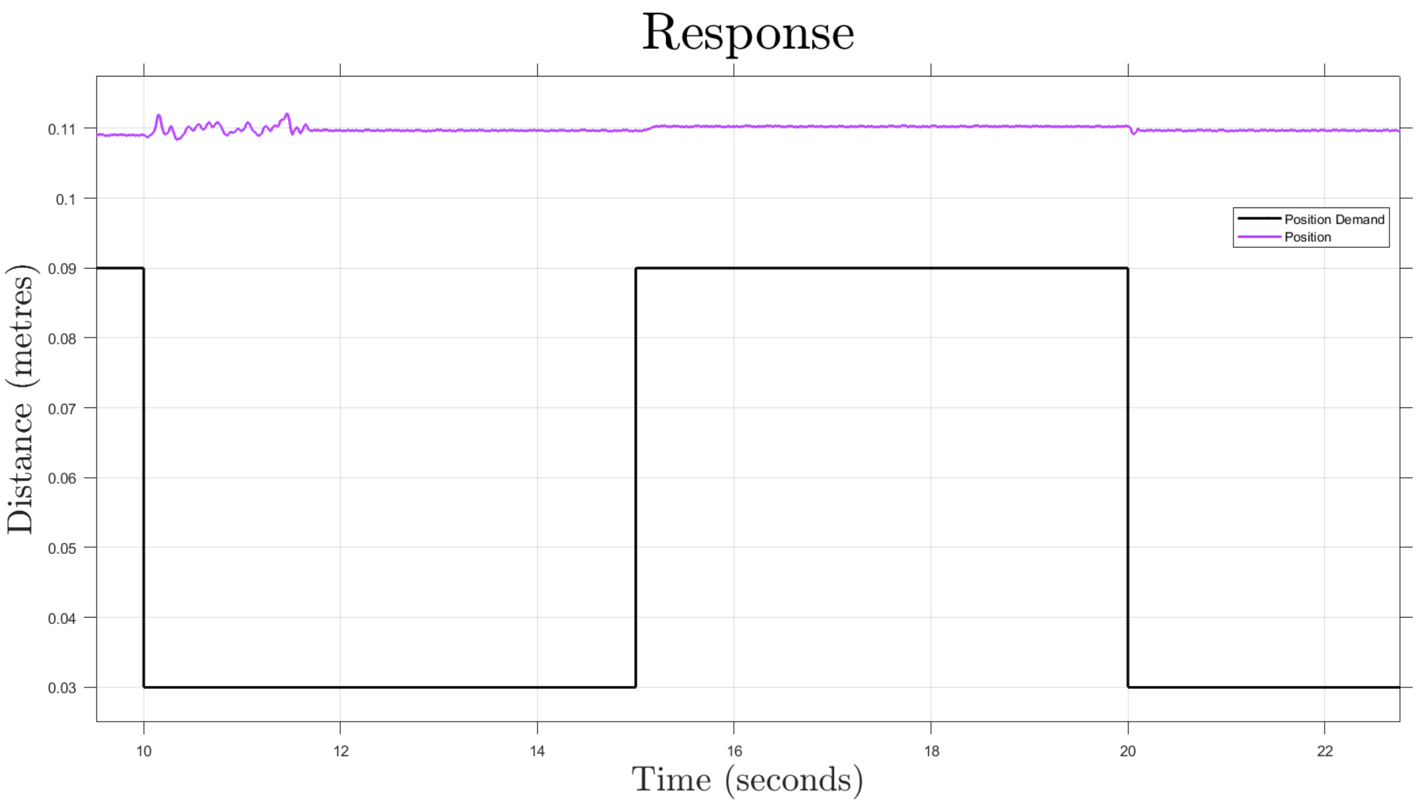

Figure 17 shows the system response with a proportional gain of 0.1 and a derivative gain of 0.9 as derived from the offline testing of the system. This response yields a 263 % overshoot and a rise time of 0.1 seconds with a very large steady state error.

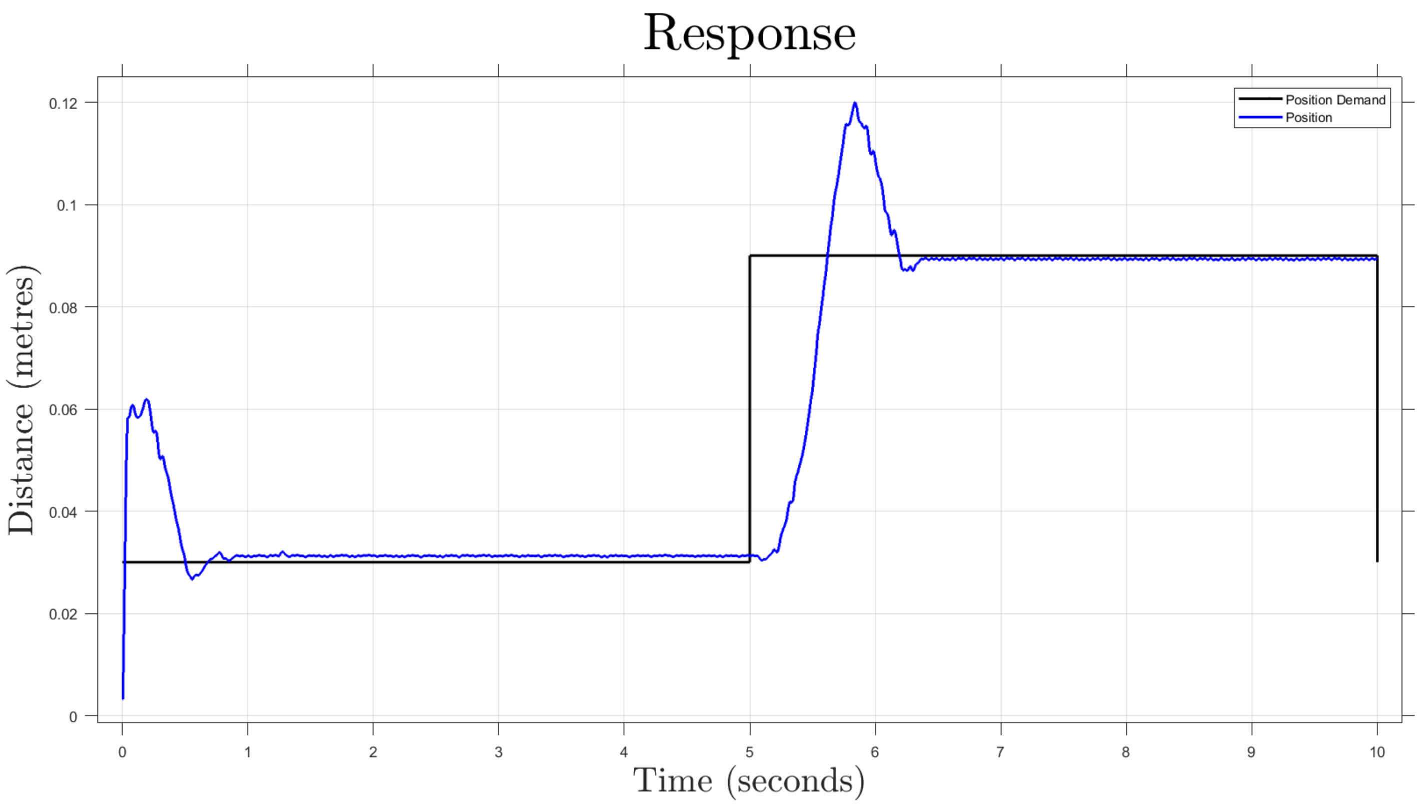

Figure 18 shows how tuning the gains to 4, 0.5 and 1 for the proportional, integral and derivative gains respectively affects the response to achieve a 33.3% overshoot, a rise time of 1.19 seconds and a steady state error of 0.001m.

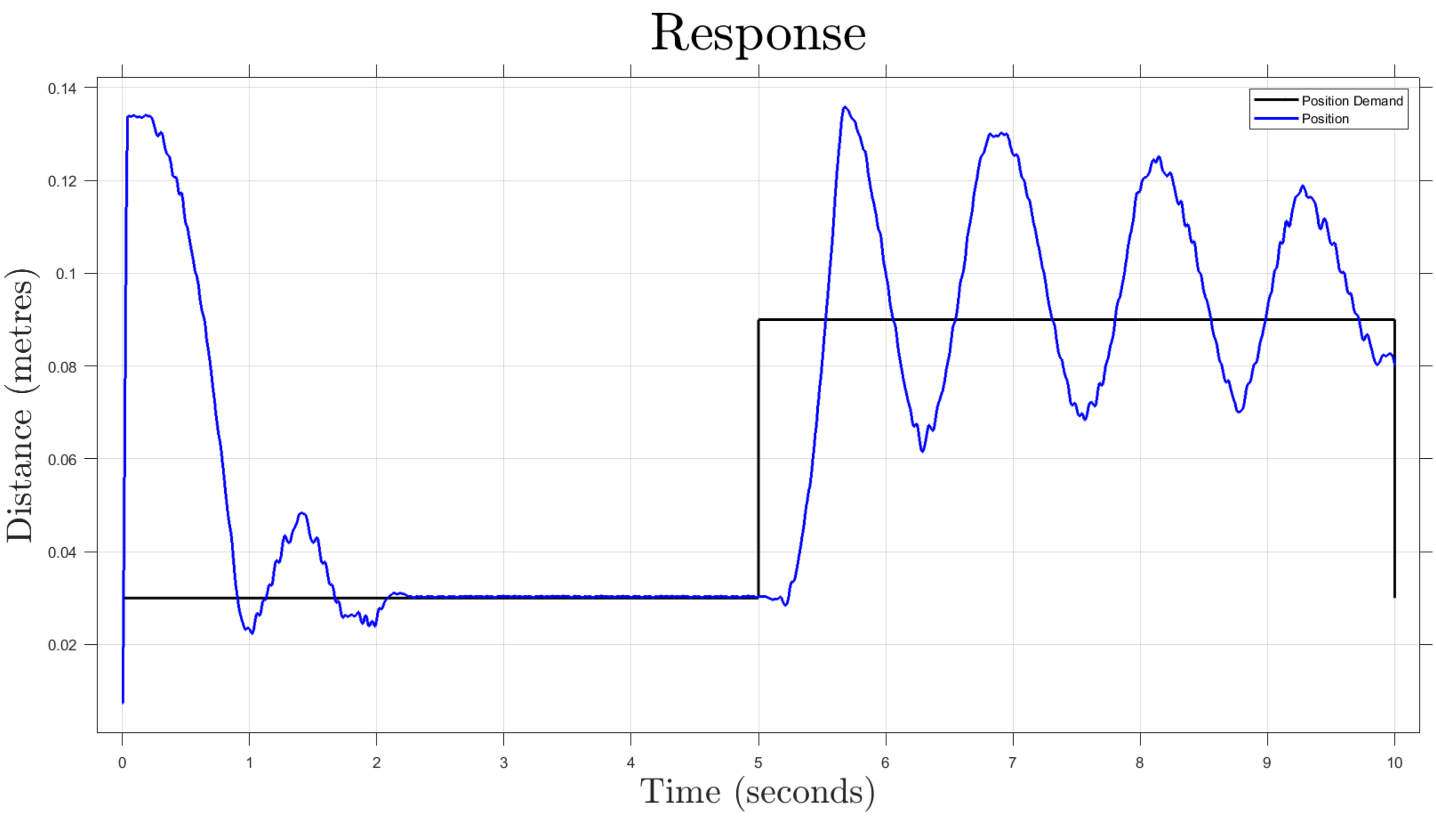

Figure 19 shows reducing the cart wheels' friction using WD40 affects the controller response. This response has a 53% overshoot and a rise time of 17.6 seconds with 0m steady state error.

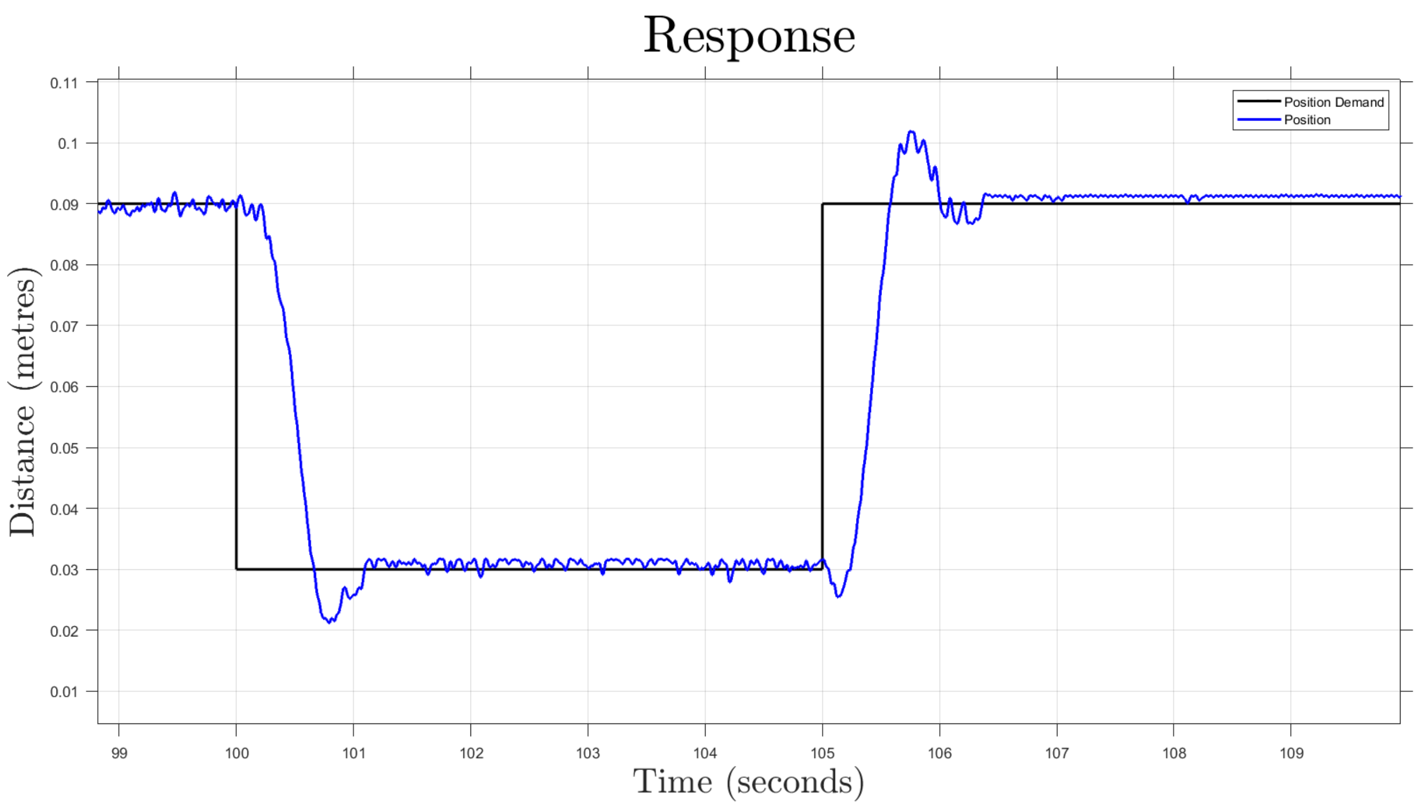

Figure 20 shows the system with tuned gains of 6, 1 and 1.6 for the proportional integral and derivative gains respectively. this response achieved a 13.3% overshoot, 1.05 seconds rise time and a 0.003m steady state error.

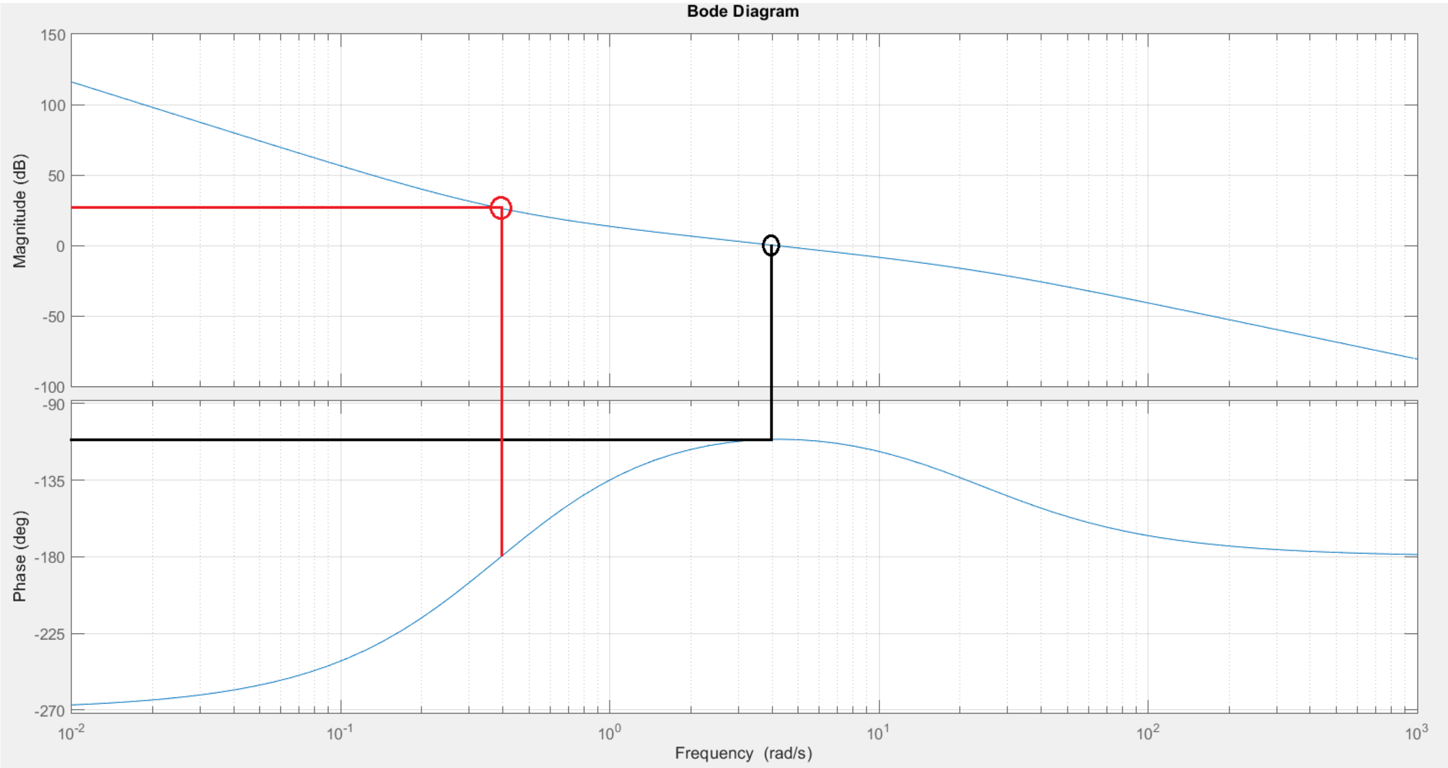

Figure 21 shows the bode plot of the system response having a gain of 29 dB and a phase margin of 75 degrees.

Discussion

Calibration and Filtering

Calibration ensures the ultrasonic sensor data reliably correlates with actual distances. The experiment found the relationship between actual and measured distances to be y=0.969x−2.1. Therefore, the sensor data needs to be offset by 2.1mm and scaled by a factor of 1.032 to match the y = x line on figure 6, representing a perfect sensor.

Real-time signal processing relies on causal filters, limited by computational power as processing must occur within one time step of the Digital Signal Processor. A low-pass filter is needed to attenuate high-frequency noise above 20 Hz as detailed in figure 7. Figure 8 shows that the Chebyshev IIR filter introduced excessive phase lag, while Figure 9 demonstrated that the Kaiser FIR filter provided the best performance with minimal phase lag even though it is more computationally demanding.

Modelling and Controller Design

A well-structured model helps define key parameters, identify constraints, detect early design flaws, and refine the system before committing to time costly simulations. To model the system, a free body diagram coupled with Newton’s second law allows us to derive the transfer function that relates the servo angle to the position of the cart, as is demonstrated from figure 10 and equation 3.

Equation 3 may be coupled with assumptions to derive equations 4 and 5. The following assumptions were made:

- Proportional Derivative controller will be used
  - Because no disturbances are considered yet, therefore no steady state errors
- Natural frequency of the system is 5 rad/s
  - Reasonable assumption

- Damping ratio is 0.7
    - Reasonable assumption
- Feedback sensor is perfect
    - Due to calibration step from figure 6

Simulation

Simulation helps optimise designs, test performance, and identify potential failures before implementation. A Simulink environment aiming to mimic real conditions was designed to test the controller. The following were considered in this environment:

- RC Servomotor speed
- Maximum and minimum rail length
- Phase lag

Figure 12 shows the impact of a slow servomotor on the response, limiting the cart's position due to physical constraints. However, the servo speed, derived from equation 1 as 26.18 rad/s, is fast enough to neglect its effect.

Figure 13 highlights how the length of the rails limits the system response, with physical limits between 200 mm and 40 mm. Therefore, the demand is never set outside this range.

Figure 14 compares the controller response with and without considering phase lag from the filter. Including phase lag results in a greater overshoot but a shorter rise time, as the second oscillation remains within 2% of the demand.

Figure 15 tunes the gains derived from the modelling section to achieve a rise time of 0.52 seconds, an overshoot of 12.6%, and a steady-state error of 0m.

System Testing

Real-time testing is essential to validate the accuracy of a model by ensuring that modelled and simulated responses reflect real-world behaviour, and that the system performs as expected before deployment. Some uncertainties in the real system may include:

- Sensor inaccuracies
    - Noise
    - Calibration imperfections
    - Surface material
    - Detection angle
    - Resolution
- External disturbances such as vibrations
- Latency in control algorithms when processing feedback data
- Unmodeled dynamics that were not captured in the simulation phase
- Friction in the cart wheels

To address these uncertainties, the steady-state error was limited by the ultrasonic sensor's resolution of 0.003 m, prompting the addition of a ±0.003 m dead zone block in the Simulink model. External vibrations were mitigated using blue tack, as mentioned in the introduction. Latency issues, previously discussed in the simulation section, were also considered and the effect of friction, shown in Figure 19, will be discussed shortly.

Implementing PD controller gains from the modelling section of this report, resulted in poor performance, with a rise time of 1.08 seconds, 73% overshoot, and a steady-state error reaching up to 0.023 m, as shown in figure 16. Figure 17 shows an even worse response using simulation-derived gains, achieving a fast 0.05-second rise time but with 263% overshoot and a large steady-state error.

Figure 18 shows how tuning the controller gains and switching to a PID controller improves the response. However, applying WD40 to reduce wheel friction increases the rise time from 1.19 to 17.6 seconds, as can be seen from figure 19. Further tuning, results in a 13.3% overshoot, a steady-state error of 0.003 m, and a rise time of 1.05 seconds, shown by figure 20. This figure also shows an initial dip in the cart's position as the system tilts faster than the cart reacts, briefly reducing the sensor-cart distance before the cart slides to the demand position.

This final response is plotted as a bode plot in figure 21 and shows the response has a phase margin of 75 degrees with a gain of 29dB. 75-degree phase margin and 29 dB gain margin suggest that the system is very stable, but it might be overdamped depending on the application.

Conclusion

In conclusion, modifications to the proposed design were made based on thorough reasoning, and a system model was developed. A simulation environment was created to test initial gains and investigate system limitations. Subsequently, a controlled test environment was used to fine-tune the controller gains for optimal performance. The final system response achieved an overshoot of 13.3%, a rise time of 1.05 seconds, a steady-state error of 0.003 m, a gain margin of 29 dB, and a phase margin of 75 degrees with proportional, integral and derivative gains of 6, 1 and 1.6 respectively. These performance metrics indicate a stable and accurate controller, with minimal oscillation and a fast response—suitable for balancing a cart on rails.

The tuned controller gains differed from those derived from the model and simulation, as real-world testing introduces uncertainties that models can't fully capture. Therefore, the most effective approach was through trial and error.

This project was an engaging exploration of control systems. However, before deploying such a controller in a real-world application, further investigation into its specific application would be necessary to better understand the performance requirements in the intended context.